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(54) Perspective correction of texture in graphics by adaptive approximation

(57) The present invention provides a computer graphics system with a texel value generator capably of generating texel values using a minimal amount of computationally intensive divisions while maintaining a selectable texel accuracy criteria along a scan line. This is accomplished by adaptively selecting divisional points which delineate the scan line segments along each scan line such that the divisional points are as widely spaced as possible without exceeding the selected texel accuracy criteria. Having selected the texel accuracy criteria, such as a texel error bound optimally spaced, divisional points along the scan lines are selected as a function of the selected accuracy criteria. In general, since texture gradients are not evenly distributed over the surface of a given object and texture variations are present between different objects of the image, it is advantageous to adaptively select division points one at a time, skipping as many pixels in between divisional points as the local texture gradient will allow. Accurate texel values are computed at these divisional points and also at the end points of the scan line. Approximate texel values are then computed for the pixels located between adjacent pair of divisional points along the scan line using a suitable scheme such as linear interpolation.

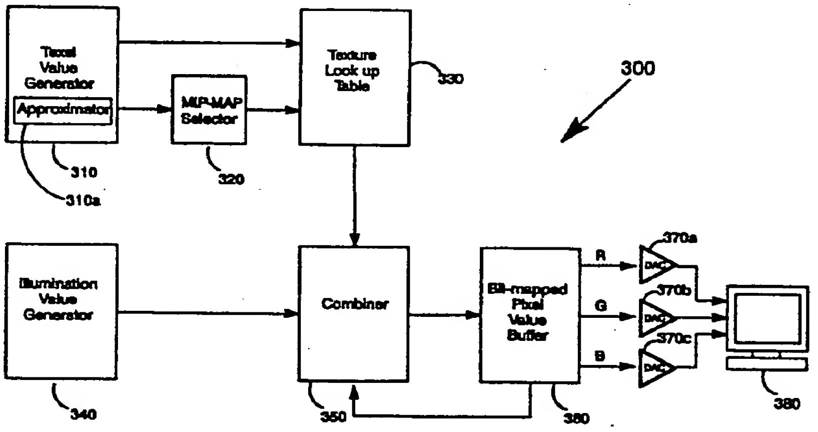

FIG. 3

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Description

The present invention relates to the field of computer graphics.

More particularly, the present invention relates to the field of texture mapping by approximation.

The application is related to U.S. patent application serial number 08/041,073, filed April 1, 1994 entitled "A Method and Apparatus for an Adaptive Textural Mapping Controller", by Yakov Kamen and Uma Sabada, and assigned to Sun Microsystems Inc., incorporated by reference herein to supplement the background discussion.

The above mentioned pending application includes a general background discussion on computer graphics and texture mapping. As such, this background section will focus on conventional texture mapping with perspective correction, methods for implementing the same, and their disadvantages.

Most conventional computer graphics systems include a display device for displaying a two dimensional image represented as a bit mapped array of evenly spaced pixels. Each pixel has a pixel value and is uniquely identified by an X-axis and a Y-axis coordinate value pair. This Cartesian X-Y coordinate representation is compatible with the majority of graphics display devices whose images are composed of a plurality of scan lines along the X-axis, e.g., cathode ray tubes (CRTs) and active/passive liquid crystal displays (LCDs).

Conventionally, texture is defined as a modulation of color, intensity, surface normal, transparency or other surface property applied to a surface of a graphics object in such a way that the pattern appears attached to the surface as the viewpoint and perspective varies. In order to realistically display three dimensional objects in a high quality two dimensional image on the display device, a perspectively correct texture mapping value, called a texel value, is generated for each pixel to modify the pixel values of the image.

Mathematical models have been employed to provide excellent image quality but they tend to be computationally intensive and very demanding of the processor on the computer graphics system. Typically, the generation of texel values include two divisional computations or their equivalent for each pixel of the image for computing the perspective correction. As is well known in the computer arts, divisional computations typically take a fairly large number of clock cycles, even when the processor is equipped with hardware dedicated to performing divisional computations. With powerful processors, such as a SPARC based processor, computing two divisions per pixel in real-time is possible but the resulting texture computations will limit the processor's ability to simultaneously support other processes. In the less powerful processors, such as Intel's 80486 SX microprocessor, real-time texture value computations requiring the two divisions per pixel can overwhelm the processor. Hence it is highly desirable to lower the computational requirements for generating a high quality perspectively corrected texture map. Typical conventional approaches reduce the total number of division computations by substituting linear interpolation for pixels at predetermined intervals between accurately computed division points.

As shown in Figures 1A and 1B, in a first pre-deterministic interpolation method, parent polygon 100 is progressively subdivided in a geometrical manner into a predetermined number of smaller child polygons 110, 120, 130, 140, ... 190. Referring to Figure 1C which shows one resulting child polygon 110 in greater detail, accurate texel values are computed for pairs of end points, e.g., end points 111a and 111z, where a scan line 111 intersects the boundaries of child polygon 110. Next, approximate texel values are computed for pixels 111b, 111c, ... 111y located between end points 111a, 111z, by linearly interpolating along scan line 111. This process is repeated for the remaining scan lines 112, 113, ... 119 of child polygon 110, and also repeated in a similar manner for every scan line of the remaining child polygons 120, 130, ... 190, until texel values for every pixel within parent polygon 100 have been computed.

Referring now to Figure 2A, a second pre-deterministic interpolation method involves subdividing each scan line, e.g., scan line 210 of a polygon 200, into scan line segments delineated by a predetermined number of divisional points 212, 213, ... 218, and end points 211, 219 is shown. For each scan line, e.g., scan line 210, accurate texel values are computed for end points 211, 219 and divisional points 212, 213, ... 218. Next, as shown in Figure 2B, texel values for the remaining pixels, 211a, 211b, ... 211z, 212a, 212b ... 212z, ..., 218a, 218b, ...218z along scan line 210 are then computed by linear interpolation. This process is repeated for the remaining scan lines 220, 230, ... 290, until texel values have been computed for every pixel in polygon 200.

In the above described conventional methods, linear interpolation of intermediate texel values can be accomplished with the following exemplary equations as applied to a polygon 2000 of Figure 2C. Polygon 2000 is a triangle specified by a triple $(x_i, y_i, z_i, w_i, u_i, v_i)$, i = 1,2,3, where (x_i, y_i) are the device coordinate of the triangle vertices, z_i are the Z depth at the vertices, w_i are the absolute value of the reciprocals of the w components of the vertices in the homogeneous clip coordinate system, and (u_i, v_i) are the two-dimensional texture coordinate of the vertices. U, V, and W are the respective linearly interpolated values of u, v, w, along the left edge of triangle 2000 between $(x_1,y_1,z_1,w_1,u_1,v_1)$ and $(x_3,y_3,z_3,w_3,u_3,v_3)$.

$$u = u_o + (x - x_o)\left(\frac{du}{dx}\right) \qquad \text{(EQ i)}$$

$$v = v_0 + (x - x_0)\left(\frac{dv}{dx}\right) \qquad \text{(EQ ii)}$$

$$w = w_0 + (x - x_0)\left(\frac{dw}{dx}\right) \qquad \text{(EQ iii)}$$

$$\text{and } u_0 = U(x_0,y_0) \qquad \text{(EQ iv)}$$

$$w_0 = W(x_0,y_0) \qquad \text{(EQ v)}$$

$$v_0 = V(x_0,y_0) \qquad \text{(EQ vi)}$$

$$\text{texture} = \text{texmap}\left(\frac{u}{q}, \frac{v}{q}\right) \qquad \text{(EQ vii)}$$

While the above described conventional methods do reduce the computational load on the processor by replacing some computationally intensive divisions with simpler linear interpolations, both conventional methods are disadvantageously inflexible, inefficient and/or produce unsatisfactory texel values. This is because pre-determining the amount of divisions works well only if texture variations between objects and texture gradients within objects over the entire image are both fairly constant. In practice, different objects in different images can have different texture properties and so no one predetermined level of scan line division will be optimal for all objects. For example, if the predetermined scan line segments are too short, i.e., the divisional points too closely spaced, an excessive amount of divisional computations will be required of the processor. Conversely, if the predetermined scan line segments are too large, grossly inaccurate approximations of texel values will result at distant pixel locations relative to the divisional points, and the overall texture quality of the image will be poor.

SUMMARY OF THE INVENTION

The present invention provides a computer graphics system with a texel value generator capably of generating texel values using a minimal amount of computationally intensive divisions, i.e., a minimal number of divisional points, while maintaining a selectable texel accuracy criteria along a scan line. This is accomplished by adaptively selecting the divisional point(s) which delineate the scan line segment(s) along each scan line such that the divisional points are as widely spaced as possible without exceeding the selected texel accuracy criteria.

Having selected the texel accuracy criteria, such as a texel error bound, the locations of the divisional points along the scan lines are computed as a function of the selected accuracy criteria In general, since texture gradients are not evenly distributed over the surface of a given object and texture variations are present between different objects of the image, it is advantageous to adaptively select division points one at a time, skipping as many pixels in between divisional points as the local texture gradient will allow. In other words, the optional number of pixels between any two adjacent divisional points varies in accordance with the local texture gradient.

Once the divisional points between the end points of a scan line have been selected, accurate texel values are computed at these divisional points and also at the end points of the scan line. Typically, two divisions or equivalent mathematical operations such a reciprocal and a multiply computation, are used to accurately computing each texel value. Next, approximate texel values are computed for the pixels located between adjacent pair of divisional points along the scan line. Suitable approximation schemes include interpolation methods such as linear interpolation.

The above described process of selecting divisional points, computing accurate texel values at these divisional points and end points, and then approximating texel values for pixels lying in between adjacent divisional points and/or end points, is repeated for each scan line overlapping the given object until all the pixel texel values for displaying the object have been computed.

The present invention will now be further described, by way of example, with reference to the accompanying drawings, in which:-

Figure 1A and 1B illustrate is a flow chart a conventional method of dividing parent polygon into a pre-determined plurality of child polygons

Figure 1C shows one of the child polygons of Figure 1B in greater detail

Figure 2A and 2B illustrate a conventional method of dividing scan lines of a polygon into groups of contiguous pixels separated by a pre-determined number of divisional points.

Figure 2C shows an exemplary triangle for illustrating linear interpolation of intermediate texel values.

Figure 3 is a block diagram illustrating a texture mapping system which includes a texel value generator of the present invention.

Figures 4A and 4B are flow charts illustrating a method for adaptively sub-dividing a scan line in accordance with the invention.

Figure 5 illustrates a polygon having an optimally sub-divided scan line.

Figure 6 is a flow chart illustrating a method for approximating texel values while satisfying a selectable texel accuracy criteria

DEFINITIONS

Pixel : an integral point located on a pre-defined coordinate system, e.g., X-Y coordinate, that has a pixel value associated with an image of a given object to be displayed.

MIP (multum in parvo) map : a pyramidal parametric data structure which allows both intra-level and inter-level trilinear interpolation schemes, e.g., in an [*u, v, d*] coordinate system, where u, v are spatial coordinates used to access points within a pyramid level, and the d coordinate is used to index and interpolate between the different levels of the pyramid.

Texel value : a texture value for mapping a corresponding pixel value so as to display texture on a given object from a given perspective.

Graphics image : a representation of a given object for display on a graphics display device.

Polygon : a mathematically convenient shaped area of interest on a given object.

DESCRIPTION OF THE PREFERRED EMBODIMENT

In the following description, numerous details provide a thorough understanding of the present invention. These details include functional blocks and exemplary texel accuracy criteria to assist a developer in implementing an efficient adaptive texel value generator in software, hardware or a combination thereof. In addition, while the texel value generator of the present invention is described with reference to a specific implementation, the invention is applicable to a wide variety of computer graphics systems and environments. In other instances, well-known circuits, structures and program code are not described in detail so as not to obscure the invention unnecessarily.

Figure 3 is a block diagram illustrating one embodiment of a texture mapping system 300 which includes a texel value generator 310 of the present invention. Texture mapping system 300 also includes a MIP-map selector 320, a texture lookup table 330, an illumination value generator 340, a combiner 350, a bit-mapped pixel value buffer 360, three digital-to-analog converters (DACs) 370a, 370b, 370c, and a display device 380.

Texel value generator 310 is coupled to texture lookup table 330, both directly and through MIP-map selector 320. The respective output nodes of texture lookup table 330 and illumination value generator 340 are coupled to combiner 350. The output node of combiner 350 is coupled to bit-mapped pixel value buffer 360. In turn, pixel value buffer 360 is coupled to the digital input nodes of DACs 370a, 370b, 370c. The analog output nodes of DACs 370a, 370b, 370c arc coupled to display device 380.

Texel value generator 310 provides u, v signals and derivative u, v signals to texture lookup table 330 and MIP selector 320, respectively. In turn, MIP selector 320 provides MIP-map select signals to texture lookup table 330. Texture lookup table 330 and illumination value generator 340 produce texture and illumination values, respectively, for combiner 350. Combiner 350 then combines these texture and illumination values, together with feedback pixel values from pixel value buffer 360, and generates composite bit-mapped pixel values for buffer 360. Finally, pixel value buffer 360 provides the respective digital RGB signals, which are converted by DACs 370a, 370b, 370c, into the corresponding analog RBG signals to display device 380.

Figure 4A and 4B are flow charts illustrating an optimal method for adaptively subdividing a scan line, the method useful in association with texel value generator 310. In one embodiment, in order to be compatible with most conventional graphics display devices, the texel values are generated along parallel scan lines which are bounded by polygons, each of which define an area of interest on a given object. Each scan line has pairs of end points where the scan line intersects the boundary of a polygon. These scan lines are decomposable into rows of pixels at integral locations in an X-Y coordinate system.

Referring to the flowchart of Figure 4A, first the end points of a scan line are adjusted to integral pixel coordinate locations by a suitable method, such as trucation or rounding up/down (step 410). If the scan line is extremely short, i.e., the span of the scan line is empty because the two integral end points have merged, then no sub-division is required (step 420). Similarly, if the span of the scan line is too short to substantially benefit from any sub-division, no sub-division is performed (step 430).

Conversely, if the span is long enough to substantially benefit from sub-division, texel value generator 310 adaptively sub-divides the span of the scan line into a minimum number of optimally spaced sub-divisions (step 440).

Referring now to the flowchart of Figure 4B, step 440 is shown in greater detail. First a suitable texel accuracy criteria is selected (step 442). In this embodiment, the texel accuracy criteria is a texel error bound. Starting at one end of the scanline, texel value generator 310 computes the location of the next integral divisional point along the scan line corresponding to the largest possible sub-divisional spacing that satisfies the selected accuracy criteria (step 444). The computation of additional divisional point(s) is repeated along the scan line until the opposite end of scan line is reached (step 446). Appendices A and B show equations and a computer program, respectively, for computing the location of divisional point(s) in the preferred embodiment.

Figure 5 illustrates a polygon 500 having a scan line 510 subdivided by division points x_2, x_3, x_4 and end points x_1, x_5. In accordance with the preferred embodiment, the error of the approximated texture values increases, reaches a maximum at or approximately at an error bound value *e* and then decreases between adjacent division/end points. Note that optimizing with respect to an error bound *e*, may result in uneven spacing as illustrated by end/division point pairs x_1 and x_2, x_2 and x_3, x_3 and x_4, and x_4 and x_5. In the preferred embodiment, the spacing *t* between adjacent divisional points are computed using the following equation:

$$t = e + 2\sqrt{e\frac{wl}{|dw|}} \qquad \text{(EQ viii)}$$

where e = the error bound
w = absolute value of the reciprocals of the *w* components

$$\text{and } dw = \frac{wr - wl}{xr - xl}$$

where *wl* = *w* value at the left divisional point

and *wr* = *w* value at the adjacent right divisional point

Note that along the span of a scan line, at a given step *t*, *dw* and *e* are constants, and *wl* changes. One suitable value for *e* is 0.5, i.e., half a pixel. By selecting *e* = 0.5, each pixel will map to within half a pixel of where it should go. Although magnified texel edges will show an occasional glitch of non-monotonicity, this can easily be corrected by decreasing the selected *e* value. Conversely, increasing the selected *e* value increases the probability of glitches since the error bound is increased.

Alternatively, equation viii can be of the form:

$$t = e + 2\sqrt{\frac{e}{|dw|}} \cdot \sqrt{wl} \qquad \text{(EQ ix)}$$

By normalizing *w* over the polygon, "*wl*" can be kept in the range [0,1]. A look up table for a square root function can be used in place of a square root computation. Further efficiency is also possible because

$$\sqrt{\frac{e}{|dw|}}$$

is constant over each polygon.

Figure 6 is a flow chart illustrating a method for approximating the texel values along the span of the scan line after the divisional point(s), if any, have been select First, texel value generator 310 computes accurate texel values at the two integral end points (step 610). Accurate texel values are also computed for the integral divisional point(s), if any, selected in step 444 (step 620). Finally, texel value generator 310 which includes an approximator 310a generates approximate texel values for the intermediate pixels located between adjacent end points and divisional point(s) of each scan line (step 630). Suitable approximation methods include interpolating techniques such as the linear interpolation using the equations EQ (i) - (ix) described above.

In some embodiments, mip map levels are also computed by approximation. Since each of the subdivided line segments defined by adjacent divisional points is affine, the MIP map level is constant across each line segment. Typically, MIP maps are decimated by a factor of 2 in *u* and *v*. Hence, MIP map levels can be approximated using the following equation:

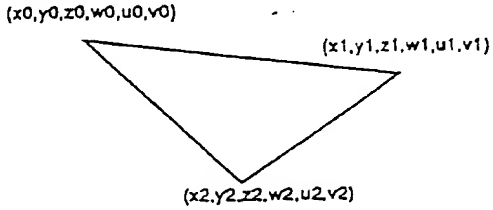
$$level = \frac{1}{2}\log_2\left(\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2\right) \qquad \text{(EQ x)}$$

While the present invention has been described in conjunction with a preferred embodiment, numerous additions and modifications are possible without departing from the spirit of the invention. Hence, the scope of the invention should be determined by the following claims.

APPENDIX A

Introduction

Let a triangle be specified by the triple (xi,yi,zi,wi,ui,vi), i=0,1,2, where (xi,yi) are the device coordinates of the triangle vertices, zi are the Z depth at the vertices, wi are the



absolute value of the reciprocals of the w components of the vertices in the homogeneous clip coordinate system, and (ui, vi) are the two-dimensional texture coordinates of the vertices.

For 2D textures of width W and height H, we define the mapping from any texture coordinate (u, v) to a point (p, q) in the half-open texel space [0,W) X [0,H) to be

$$\begin{aligned} p &= (u-\lfloor u\rfloor)\times W \\ q &= (v-\lfloor v\rfloor)\times H \end{aligned} \qquad \text{(EQ 1)}$$

This is slightly different from the XGL and OpenGL formulation that maps u=1 or v=1 to W-1 or H-1, respectively. (EQ 1) is preferable since it has a simple mapping to fixed point arithmetic. For example, to perform the linear interpolation

$$u = u0+n\times du \qquad \text{(EQ 2)}$$

and map u to texel p, we would compute

$$\begin{aligned} U0 &= u0-\lfloor u0\rfloor \\ DU &= du-\lfloor du\rfloor \end{aligned} \qquad \text{(EQ 3)}$$

$$U = U0+n\times DU \qquad \text{(EQ 4)}$$

$$p = U\times W \qquad \text{(EQ 5)}$$

where U0 and DU are considered to be unsigned binary fractions 0.32, (EQ 4) is computed in 32 bit arithmetic ignoring overflow, and (EQ 5) is computed to as many fractional bits as desired (e.g., (U*W)>>32 gives nearest neighbor mapping.)

To convert from XGL or OpenGL texture coordinates to ours, it suffices to apply a scale factor of slightly less than one in u and v.

An *affine* texturing results when the wi values are ignored and the (ui, vi) are linearly interpolated over the triangle. A *perspective* texturing results when (ui * wi, vi * wi, wi) are linearly interpolated over the triangle, and the texture coordinates (u, v) obtained by computing (ui wi / wi, vi wi / wi) per pixel. In other words, you multiply ui and wi, and interpolate that product. For each pixel, you divide the interpolated product by the interpolated wi.

Clearly, the two are equivalent if wi is constant over the triangle. Also clearly, the difference between the affine and perspective mapping increases as the ratio min(wi) / max(wi) decreases.

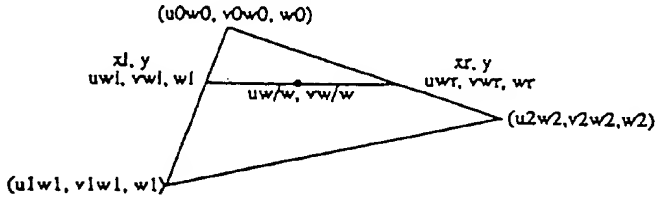
Due to the linearity of the operation in (EQ 1), for an affinely textured triangle, the (ui, vi) values as well as the du/dy, dv/dy, du/dx, dv/dx values over the triangle can all be mapped to the range [0,1) via the operation $u - \lfloor u \rfloor = Frac(u)$ applied once at the vertices. Specifically, it is obvious that

$$Frac(u0 + n \times du) = Frac(Frac(u0) + n \times Frac(du)) \quad \text{(EQ 6)}$$

This is why (EQ 3) and (EQ 4) work.

Adaptive Subdivision

Perspective texturing interpolates uw, vw, and w, instead of u and v. Thus, along the left edge of the triangle below, we interpolate, e.g., uwl linearly between u0 x w0 and u1 x w1. Across the scan line, we linearly approximate, e.g., uw between uwl and uwr.



There would be at most a minor gain in performance in doing an adaptive approximation along the triangle edges; we would save only the computation of wl and wr at the cost of the extra work in doing the approximation. However, across the scan line, the work to be saved is the two divides per pixel (or one reciprocal and two multiplies).

The approximation we wish to pursue is a piecewise-linear approximation to the perspective texturing (uw/w, vw/w) across the scan line at y (see the figure.) We wish to subdivide the scan line into segments such that:

- Each segment has the same maximum error
- There are as few segments as possible

As our error criteria, we choose pixel error. That is, we want a particular texel at (u,v) to map linearly to (x',y') where (x'y') is close enough to the correct perspective location (x,y). It is incorrect to choose texel error if one wants to use a constant error bound.

Since we are operating on a scan line only, we need consider only x as varying. We will derive the below using x and u only; we will discover a surprising result that obviates the need to derive the x and v relationship.

We have the following equations:

$$T(x) = \frac{x - xl}{xr - xl} \quad \text{(EQ 7)}$$

which maps the pixel location x in the range [xl, xr] to a value t in the range [0,1]:

$$UL(t) = ul + t(ur - ul) \quad \text{(EQ 8)}$$

which is the linear interpolation of u between ul and ur;

$$UP(t) = \frac{ulwl + t(urwr - ulwl)}{wl + t(wr - wl)} \qquad \text{(EQ 9)}$$

which is the perspective interpolation of u between ul wl/wl and ur wr/wr; and finally

$$M(u) = Wu \qquad \text{(EQ 10)}$$

which maps u in the range [0,1) to the corresponding texture map pixel horizontal coordinate. W is the width of the texture map. If a nearest neighbor mapping is desired; then one takes the floor of M.

The pixel error then becomes, given the horizontal coordinate m,

$$err(m) = T^{-1}(UL^{-1}(M^{-1}(m))) - T^{-1}(UP^{-1}(M^{-1}(m))) \qquad \text{(EQ 11)}$$

the left term is the inverse of the affine approximation, the right term is the perspective inverse.

Now, let's find the value of m for which the error is maximal.

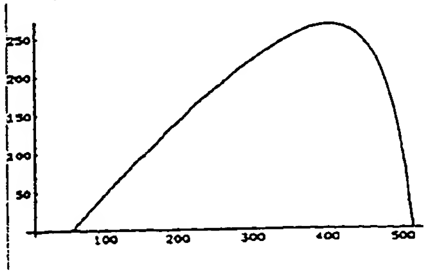
We have

$$\begin{aligned} T^{-1}(t) &= xl + t(xr - xl) \\ UL^{-1}(u) &= \frac{u - ul}{ur - ul} \\ UP^{-1}(u) &= \frac{(ulwl - uwl)}{(ulwl + uwr - urwr - uwl)} \\ M^{-1}(m) &= \frac{m}{W} \end{aligned} \qquad \text{(EQ 12)}$$

Expanding out err(m) using the above and simplifying, we have

$$err(m) = \frac{(Wul - m)(Wur - m)(wr - wl)(xr - xl)}{W(ur - ul)(mwr + Wulwl - mwl - Wurwr)} \qquad \text{(EQ 13)}$$

Examining the shape of the error curve with Mathematica, we see that it is a convex curve with a well-defined maxima or minima. A sample curve is in the following figure. The next step is differentiating with respect to m:



$$\frac{d}{dm}err(m) = \frac{(wr-wl)(xr-xl)(m^2(wr-wl)-2mW(urwr-ulwl)+W^2(ur^2wr-ul^2wl))}{W(ur-ul)(mwr+Wulwl-mwl-Wurwr)^2} \quad \text{(EQ 14)}$$

In the denominator, W is always nonzero. We assume, for the moment, that $ul \neq ur$. That assumption also ensures that the term

$$mwr+Wulwl-mwl-Wurwr = m(wr-wl)+Wulwl-Wurwr \quad \text{(EQ 15)}$$

is nonzero, as well. To see that, rewrite Wul as ml and Wur as mr. Then, for it to be zero, we must have

$$wlml-wrmr = m(wl-wr) \quad \text{(EQ 16)}$$

Since m is in the range [ml, mr], we can write m as ml + t(mr-ml); substituting in the above and simplifying, we get

$$wr(ml-mr) = t(wl-wr)(mr-ml) \quad \text{(EQ 17)}$$

Since $ml \neq mr$, that reduces to

$$t = \frac{wr}{(wr-wl)} \quad \text{(EQ 18)}$$

Now we have wl, wr > 0, therefore t > 1, which contradicts m being in the range [ml, mr]. Therefore, (EQ 15) is nonzero.

Thus, we can find the maximum by equating the numerator of (EQ 14) to zero, and solving for m.

This gives us the two solutions

$$m_{maxerr} = \left(\frac{W(ul\sqrt{wl}-ur\sqrt{wr})}{\sqrt{wl}-\sqrt{wr}}, \frac{W(ul\sqrt{wl}+ur\sqrt{wr})}{\sqrt{wl}+\sqrt{wr}}\right) \quad \text{(EQ 19)}$$

Since wl can equal wr, the first solution is not the one we seek. Now, we substitute the second value back into err(m) from (EQ 13) to get the remarkable value for the error

$$err(m_{maxerr}) = (xr-xl)\frac{\sqrt{wr}-\sqrt{wl}}{\sqrt{wr}+\sqrt{wl}} \quad \text{(EQ 20)}$$

An alternative formulation of the above is to let $\alpha = \frac{min(wl,wr)}{max(wl,wr)}$ and rewrite it as

$$err(m_{maxerr}) = (xr-xl)\frac{1-\sqrt{\alpha}}{1+\sqrt{\alpha}} \quad \text{(EQ 21)}$$

The amazing thing about the above is that it is independent of u and the texture map width W! To paraphrase, the pixel error in using affine texturing instead of perspective texturing depends only on the width of the span and the ratio of the w values at the span ends.

Going back to our original assumption that $ul \neq ur$, we can now see that if they are equal, no subdivision is needed (the whole scan line is the same texel). In that case, for simplicity we choose to subdivide anyway rather than check for that exception.

Now let's calculate the pixel value in the range [xl, xr] for which the maximum error occurs. There are two ways to map m_{maxerr} to x, via

$$T^{-1}(UL^{-1}(M^{-1}(m))) \qquad \text{(EQ 22)}$$

or via

$$T^{-1}(UP^{-1}(M^{-1}(m))) \qquad \text{(EQ 23)}$$

The correct inverse to use is (EQ 23), as we want the correct x, not the approximate x. This gives the point of maximum error as

$$x_{maxerr} = \frac{xl\sqrt{wr} + xr\sqrt{wl}}{\sqrt{wl} + \sqrt{wr}} \qquad \text{(EQ 24)}$$

Now, we are ready to solve the problem as posed originally. Let

$$\begin{aligned} xt &= xl + t \\ dw &= \frac{wr - wl}{xr - xl} \\ wt &= wl + t\,dw \end{aligned} \qquad \text{(EQ 25)}$$

for t=0, 1, ..., xr-xl. Then we can rewrite (EQ 20) as

$$err(t) = t\frac{\sqrt{wt} - \sqrt{wl}}{\sqrt{wt} + \sqrt{wl}} \qquad \text{(EQ 26)}$$

(EQ 26) gives us the maximum error of an affine approximation if we subdivide at xt. Now let us solve for t given a fixed error e. We have the two solutions

$$t = e \pm 2\sqrt{e\frac{wl}{dw}} \qquad \text{(EQ 27)}$$

Since t must increase as e increases, we pick the + branch. If dw < 0, we choose the negative sign for e to maintain a real radical; we get the same effect by choosing Abs(dw] instead. Therefore, the point to subdivide a span given a desired maximum error e > 0 is

$$t = e + 2\sqrt{e\frac{wl}{|dw|}} \qquad \text{(EQ 28)}$$

Note that as we step along the span, t at a time, dw and e are constant, and only wl changes.

A good value to use for e is 0.5, e.g., half a pixel. That means that each texel will map to within half a pixel of where it should go. Magnified texel edges will show an occasional glitch of nonmonotonicity. Increasing the error increases the glitch probability and also increases the maximum magnitude. See the figures for examples. Since we actually are subdividing across a scan line, we want to subdivide at pixel boundaries. We simply adjust t to the next pixel.

Determining triangle affineness

How does one determine if a triangle is sufficiently small (or the w values are close enough) that the entire triangle can be textured affinely?

We use (EQ 21). Since we did not, in fact, make any assumptions about the direction of the span, we can conclude that a triangle is affine if the error is $err(m_{maxerr}) \le 0.5$ for all segments within the triangle, with the (xr - xl) term replaced by the length of the segment.

We can calculate the maximum possible length by using the max norm, or the max of the width or height of the triangle bounding box. To calculate the maximum value of the w expression, let $\alpha = \frac{min(w0, w1, w2)}{max(w0, w1, w2)}$ and expand (EQ 21) to

$$err_{triangle} \le max(width, height)\frac{1-\sqrt{\alpha}}{1+\sqrt{\alpha}} \qquad \text{(EQ 29)}$$

We can lookup the square root expression in a table indexed between (0,1]. The cost of determining triangle affineness is the cost of calculating the bounding box, the minimum ratio, a table lookup and a multiply.

We have not investigated more accurate estimations of affineness. One possible approach is to observe that the maximum error must occur along a segment whose endpoints lie on the triangle boundary.

Mip map level calculation

Since each of the subdivided segments is affine, the scale is constant across that segment. Thus, the mipmap level is also constant across the segment.

A quick way to estimate the scale is to assume that the texture is uniformly scaled and just compute the euclidean norm across the scan line

$$scale = \sqrt{\left(\frac{du}{dx}\right)^2+\left(\frac{dv}{dx}\right)^2} \qquad \text{(EQ 30)}$$

Since the mipmaps typically used are decimated by a factor of 2 in u and v, the corresponding mipmap level to choose is $\log 2(scale)$, or

$$level = \frac{1}{2}\log_2\left(\left(\frac{du}{dx}\right)^2+\left(\frac{dv}{dx}\right)^2\right) \qquad \text{(EQ 31)}$$

Note that the ilogb function is a quick approximation to $\log_2$, as one will be using an integral level anyway.

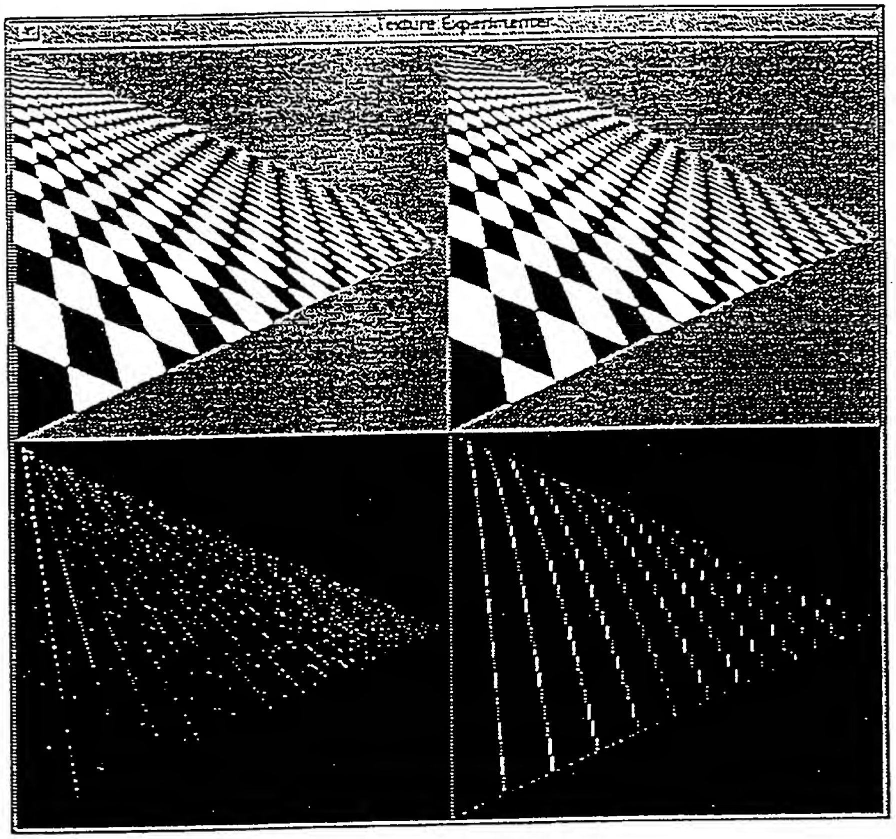


FIGURE 1.

Difference of perspective texturing and adaptive subdivision affine approximation. The correct texture is in the upper left, and the adaptive texture in the upper right. The lower left is the XOR of the two upper images. The lower left shows the adaptive subdivision. The texture is drawn affinely along each horizontal segment between the staircase curves. The error cutoff is 0.5.

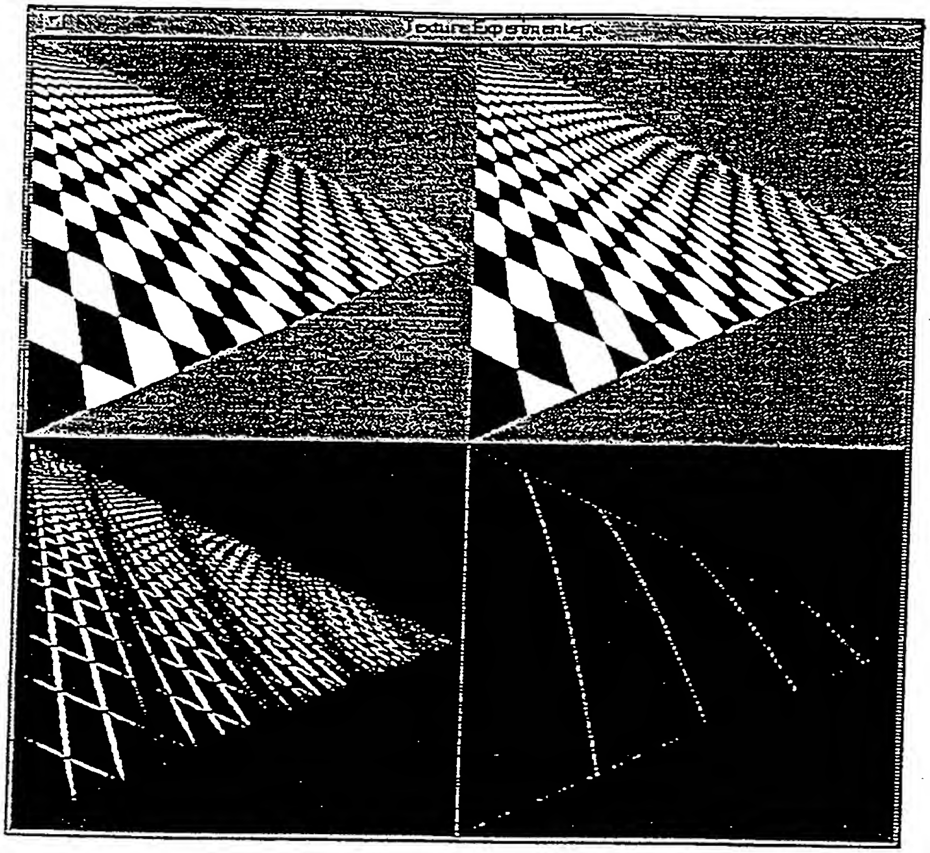


FIGURE 2.

The only difference between this and FIGURE 1. is that the error cutoff is now 4.0. There are fewer subdivisions per scan line. If the texture map were an image of a face (fewer high-frequency spatial components), then the approximation would still be nearly indistinguishable from the exact texturing.

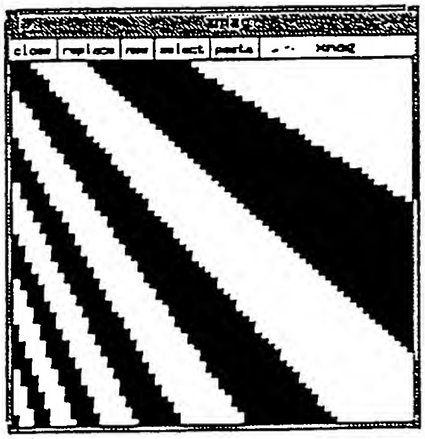


FIGURE 3. Magnified image of exact perspective texturing

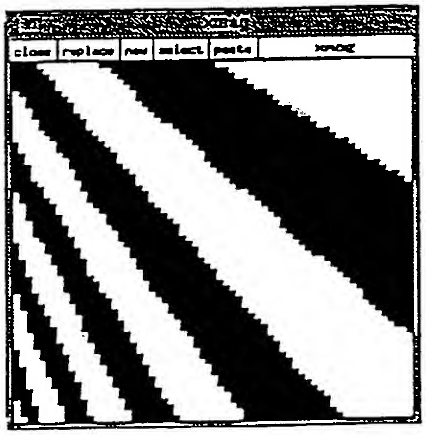


FIGURE 4. Magnified image of approximate texturing. The effect of the 0.5 pixel error threshold is clearly visible along the edges of the mapped texture.

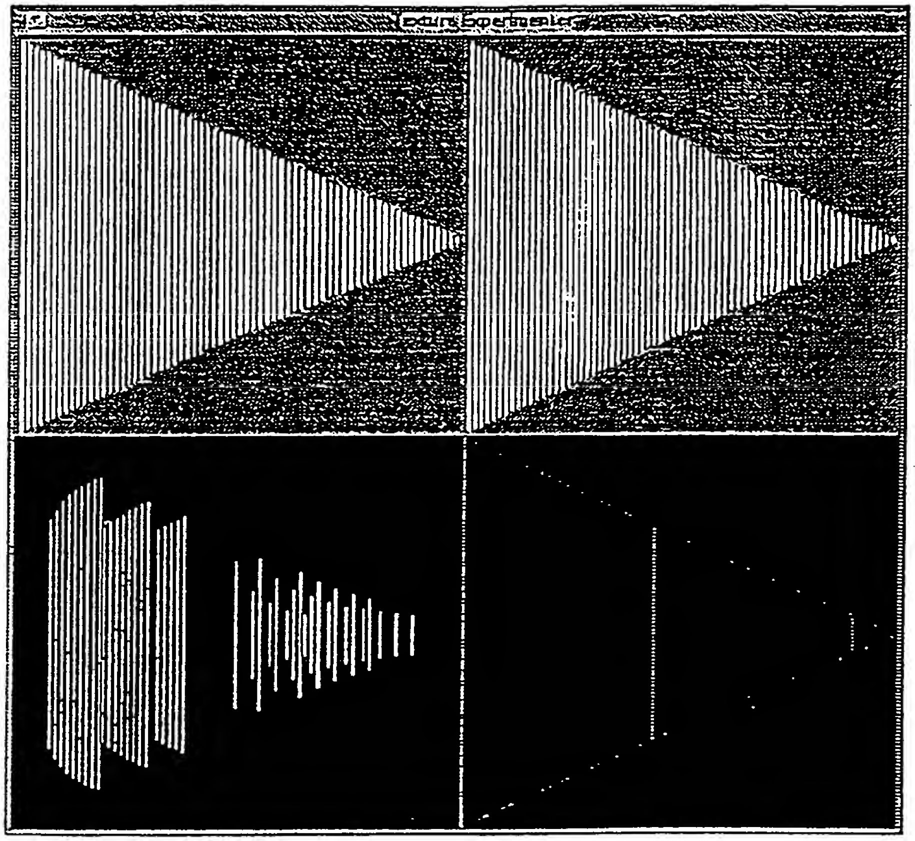
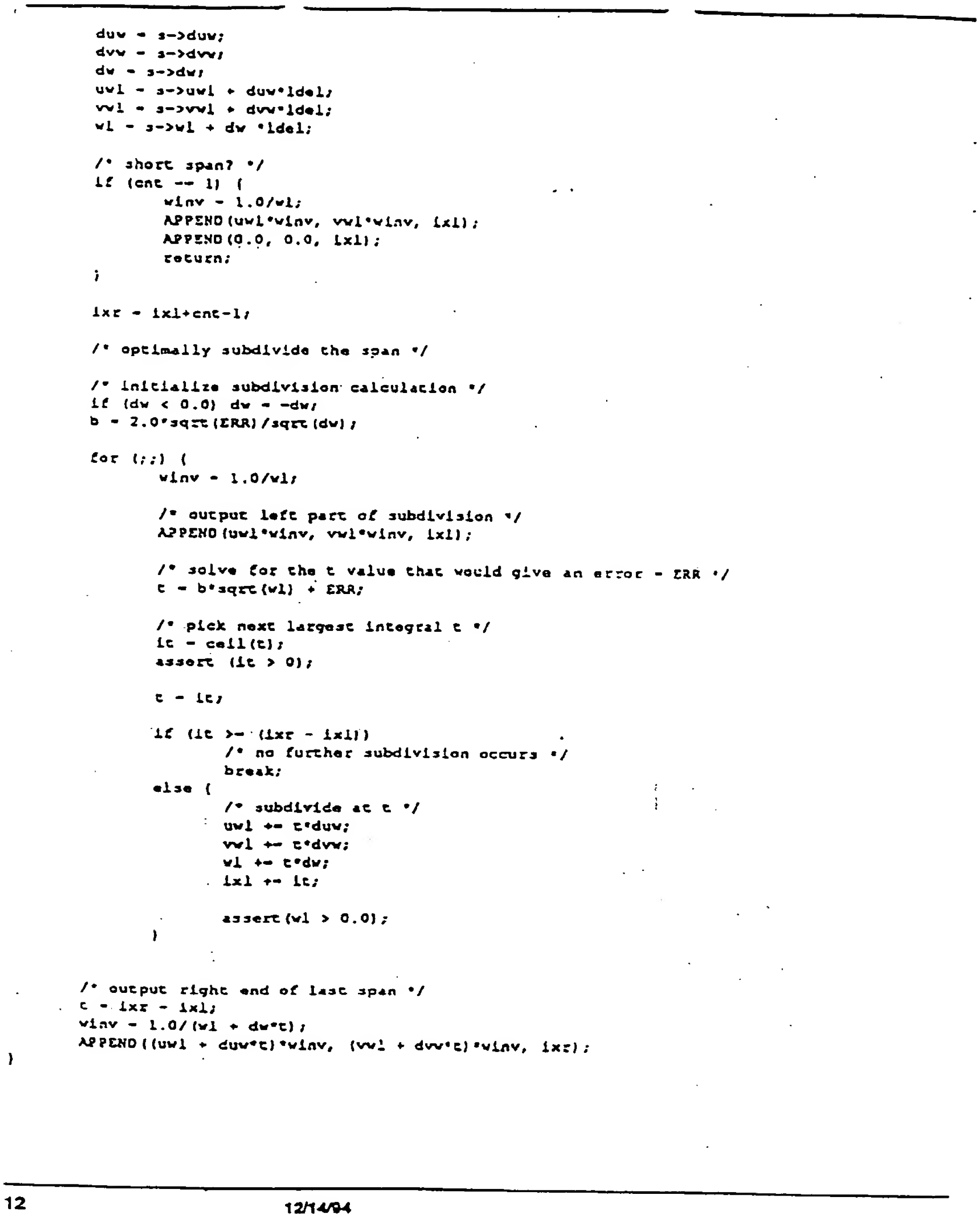


FIGURE 5.

Here, a moire pattern is used to magnify the effect of an error threshold of 0.5. The correct texturing is on the left. The two patterns become identical at an error threshold of 0.010.

APPENDIX B

```
/* Exemplary routine to subdivide a span to approximate a persp-ctive transform */

/* pixel error threshold. An experimentally-determined good value */
#define ERR 0.5

#define MAXKNOTS (100)

struct subdiv {
/*
        uwl, vwl, wl: u*w, v*w, w at left of span. Note that it is u*w, v*w,
                and w that are linearly interpolated down the edges.
        duw, dvw, dw: d(uw)/dx, d(vw)/dx, dw/dx, which are constant over
                the triangle.
        xl, xr: the span in fract pixels. we subdivide this span at
                integral pixel coords.
*/
        float  uwl;
        float  vwl;
        float  wl;
        float  duw;
        float  dvw;
        float  dw;
        float  xl;
        float  xr;
        int n;                     /* # of values here */
                                   /* if n == 0, empty span.
                                    for n >= 2, last span extends from
                                    x to x, inclusive. Initial spans
                                    extend from x to x-1 */
        struct {
                int x; /* x of start of subspan */
                float u;/* u at start of subspan */
                float v;/* v at start of subspan */
        } knots[MAXKNOTS];
};

#define APPEND(uu,vv,xx) do { \
        assert(s->n>=0 && s->n<MAXKNOTS); \
        s->knots[s->n].u = uu; \
        s->knots[s->n].v = vv; \
        s->knots[s->n].x = xx; \
        s->n++; } while(0)

static void
subdiv(struct subdiv *s)
{
        int    ixl, ixr, cnt, it;
        float  ldel, duw, dvw, dw, uwl, vwl, wl, winv, b, t;

        /* initialize to no subdivisions */
        s->n = 0;

        /* adjust endpoints to integral pixels */

        /* ceil() and ceil()-1 rule for left and right polygon edges */
        ixl = ceil (s->xl);
        cnt = ceil(s->xr) - ixl;

        /* empty span? */
        if (cnt <= 0)
                return;

        ldel = s->xl - ixl;
```



```
        duw = s->duw;
        dvw = s->dvw;
        dw = s->dw;
        uwl = s->uwl + duw*ldel;
        vwl = s->vwl + dvw*ldel;
        wl = s->wl + dw *ldel;

        /* short span? */
        if (cnt == 1) {
                winv = 1.0/wl;
                APPEND(uwl*winv, vwl*winv, ixl);
                APPEND(0.0, 0.0, ixl);
                return;
        }

        ixr = ixl+cnt-1;

        /* optimally subdivide the span */

        /* initialize subdivision calculation */
        if (dw < 0.0) dw = -dw;
        b = 2.0*sqrt(ERR)/sqrt(dw);

        for (;;) {
                winv = 1.0/wl;

                /* output left part of subdivision */
                APPEND(uwl*winv, vwl*winv, ixl);

                /* solve for the t value that would give an error = ERR */
                t = b*sqrt(wl) + ERR;

                /* pick next largest integral t */
                it = ceil(t);
                assert (it > 0);

                t = it;

                if (it >= (ixr - ixl))
                        /* no further subdivision occurs */
                        break;
                else {
                        /* subdivide at t */
                        uwl += t*duw;
                        vwl += t*dvw;
                        wl += t*dw;
                        ixl += it;

                        assert(wl > 0.0);
                }
        }

        /* output right end of last span */
        t = ixr - ixl;
        winv = 1.0/(wl + dw*t);
        APPEND((uwl + duw*t)*winv, (vwl + dvw*t)*winv, ixr);
}
```

12 12/14/94

Claims

1. In a computer graphics system for generating and displaying images of objects on a display device, the images including pixels having pixel values, the graphics system including elements for modifying the images displayed on the display device, a method for efficiently computing texel values so as to display texture on a given object from a given perspective, said method comprising the steps of:
   adaptively selecting a plurality of divisional points between a pair of end points along a scan line; and
   approximating a plurality of intermediate texel values for a corresponding plurality of pixels located between adjacent divisional points of said scan line.

2. The method of claim 1 wherein said approximating step includes an interpolating step.

3. The method of claim 2 wherein said interpolating step is a linear interpolating step.

4. The method of claim 1 wherein said adaptive selection step comprises the steps of:
   selecting a texel accuracy criteria; and
   computing the locations of said divisional points located along said scan line as a function of said selected texel accuracy criteria.

5. The method of claim 4 wherein said texel accuracy criteria is a texel error bound.

6. The method of claim 5 wherein said texel error bound is a constant *e*, and a step *t* between adjacent divisional points is computed using an equation:

$$t = e + 2\sqrt{e\frac{wl}{|dw|}}$$

7. The method of claim 1 further comprising the steps of:
   approximating a second plurality of intermediate texel values for a second corresponding plurality of pixels located between one of said end points and an adjacent divisional point of said scan line.

8. The method of claim 7 wherein said approximating step includes an interpolating step.

9. The method of claim 8 wherein said interpolating step is a linear interpolating step.

10. The method of claim 1 further comprising the step of:
    accurately computing a corresponding plurality of texel values at said divisional points and said end points of said scan line.

11. The method of claim 1 further comprising the step of using said intermediate texel values to modify said pixel values of said pixels.

12. The method of claim 11 wherein said modifications of said pixel values includes a step of computing MIP map level values for said intermediate texel values using an equation:

$$level = \frac{1}{2}\log_2\left(\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2\right)$$

13. A method of providing a texel generator for efficiently computing texel values so as to display texture on a given object from a given perspective, said method comprising the steps of:
    providing a calculator for adaptively selecting a plurality of divisional points along a scan line; and
    providing an approximator for approximating a plurality of intermediate texel values for a corresponding plurality of pixels located between adjacent divisional points of said scan line.

14. The method of claim 13 wherein said approximator includes an interpolator.

15. The method of claim 14 wherein said interpolator is a linear interpolator.

16. The method of claim 13 wherein said calculator includes an element for performing the steps of:

selecting a texel accuracy criteria; and

computing the locations of said divisional points located along said scan line as a function of said selected texel accuracy criteria.

17. The method of claim 16 wherein said texel accuracy criteria is a texel error bound.

18. The method of claim 17 wherein said texel error bound is a constant *e*, and a step *t* between adjacent divisional points is computed using an equation:

$$t = e + 2\sqrt{e\frac{wl}{|dw|}}$$

19. The method of claim 13 wherein said approximator also approximates a second plurality of intermediate texel values for a second corresponding plurality of pixels located between an end point and an adjacent divisional point of said scan line.

20. The method of claim 13 wherein said calculator includes an element for accurately computing a plurality of texel values at said divisional points said end points of said scan line.

21. The method of claim 13 further comprising the step of providing a combiner for using said intermediate texel values to modify said pixel values of said pixels.

22. The method of claim 21 wherein said modification of said pixel values includes a step of computing MIP map level values for said intermediate texel values using an equation:

$$level = \frac{1}{2}\log_2\left(\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2\right)$$

23. An adaptive texel value generator useful in association with a computer graphics system for generating, modifying and displaying an image of objects on a display device, the image including pixels having pixel values, said texel value generator comprising:

a calculator for adaptively computing the locations of a plurality of divisional points along a scan line as a function of a selected texel accuracy criteria; and

an approximator coupled to said calculator, for computing a plurality of intermediate texel values for a corresponding plurality of pixels located between adjacent divisional points of said scan line.

24. The generator of claim 23 further comprising a memory element coupled to said calculator, for storing said selected texel accuracy criteria.

25. The generator of claim 24 wherein said stored texel accuracy criteria is a texel error bound.

26. The generator of claim 25 wherein said texel error bound is a constant *e*, and a step *t* between adjacent divisional points is computed using an equation:

$$t = e + 2\sqrt{e\frac{wl}{|dw|}}$$

27. The generator of claim 23 wherein said approximator includes an interpolator for interpolaring between texel values of said adjacent divisional points.

28. The generator of claim 25 wherein said interpolator is a linear interpolator.

29. The generator of claim 23 wherein said calculator includes a divider for accurately computing texel values for said divisional points.

30. The generator of claim 23 wherein said pixel values are modified by computing MIP map levels of said intermediate texel values using an equation:

$$level = \frac{1}{2}\log_2\left(\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2\right)$$

31. A texture mapping system useful in association with a computer graphics system for generating, modifying and displaying an image of objects on a display device, the image including pixels having pixel values, said graphics system comprising:

a calculator for adaptively computing the locations of a plurality of divisional points along a scan line as a function of a selected texel accuracy criteria;

an approximator coupled to said calculator, for computing a plurality of intermediate texel values for a corresponding plurality of pixels located between adjacent divisional points of said scan line; and

a combiner coupled to said approximator for modifying said pixel values with said intermediate texel values.

32. The mapping system of claim 31 further comprising a memory element coupled to said calculator, for storing said selected texel accuracy criteria.

33. The mapping system of claim 32 wherein said stored texel accuracy criteria is a texel error bound.

34. The mapping system of claim 33 wherein said texel error bound is a constant *e*, and a step *t* between adjacent divisional points is computed using an equation:

$$t = e + 2\sqrt{e\frac{wl}{|dwl}}$$

35. The mapping system of claim 31 wherein said approximator includes an interpolator for interpolating between texel values of said adjacent divisional points.

36. The mapping system of claim 35 wherein said interpolator is a linear interpolator.

37. The mapping system of claim 31 wherein said calculator includes a divider for accurately computing texel values for said divisional points.

38. The mapping system of claim 31 wherein said pixel values are modified by computing MIP map levels of said intermediate texel values using an equation:

$$level = \frac{1}{2}\log_2\left(\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dx}\right)^2\right)$$

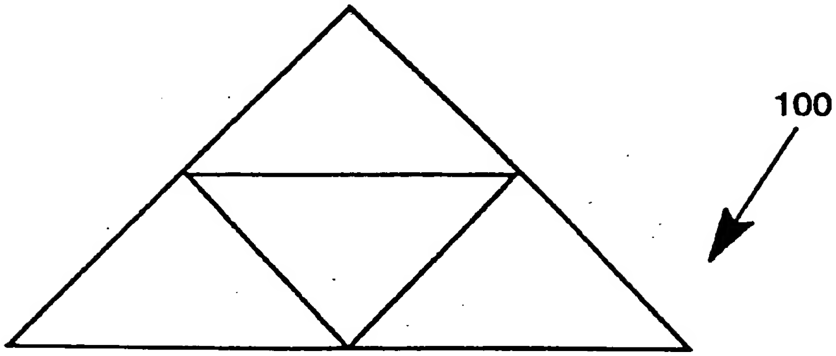


FIG.1A

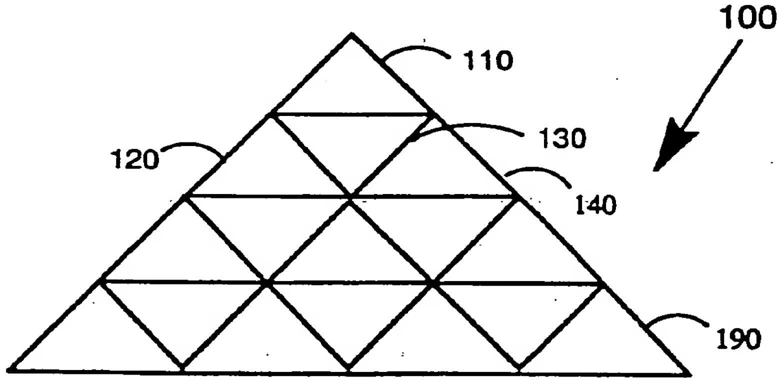


FIG.1B

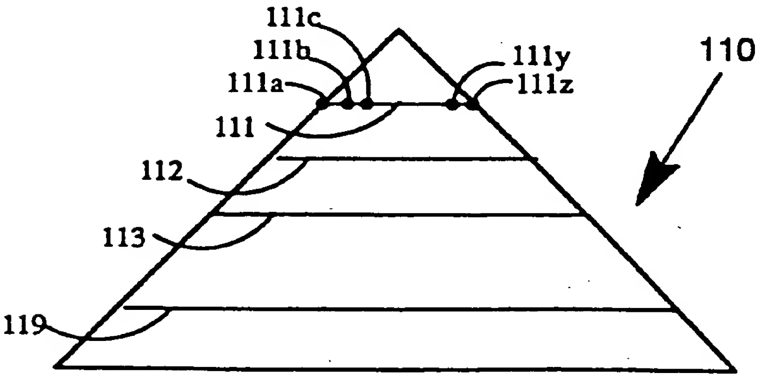
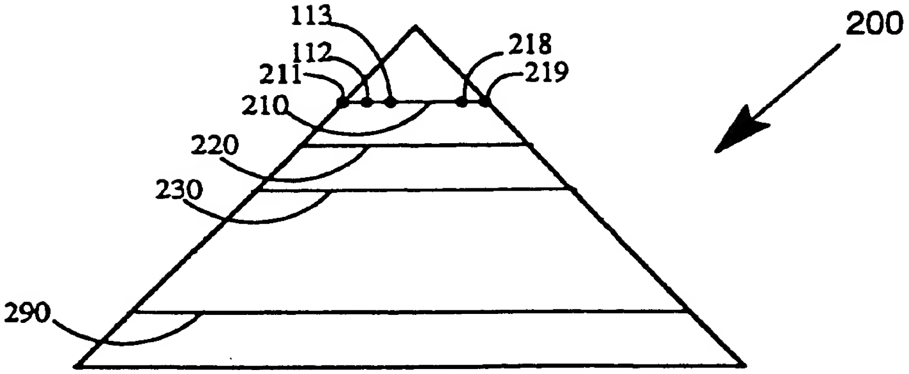
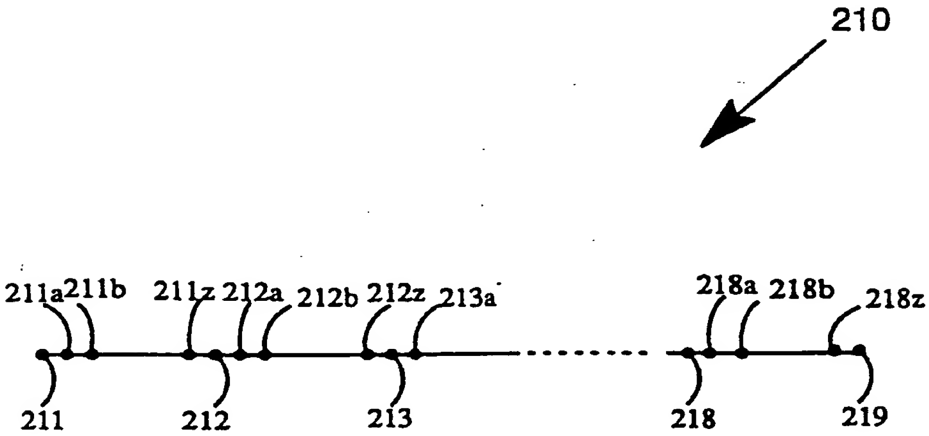


FIG.1C



113
112
211
210
220
230
290
218
219
200

FIG.2A



210
211a
211b
211z
212a
212b
212z
213a
218a
218b
218z
211
212
213
218
219

FIG.2B

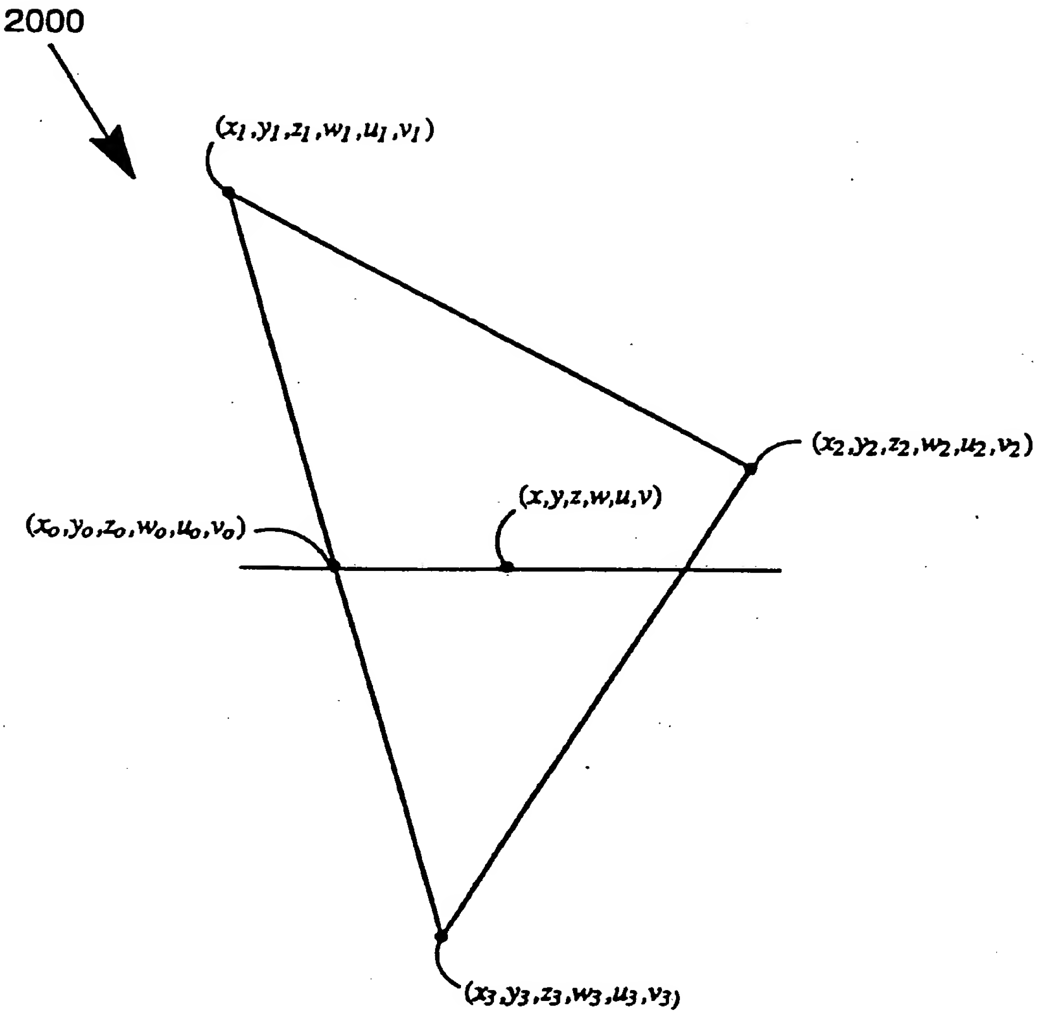

2000
$(x_1,y_1,z_1,w_1,u_1,v_1)$
$(x_2,y_2,z_2,w_2,u_2,v_2)$
(x,y,z,w,u,v)
$(x_0,y_0,z_0,w_0,u_0,v_0)$
$(x_3,y_3,z_3,w_3,u_3,v_3)$

FIG. 2C

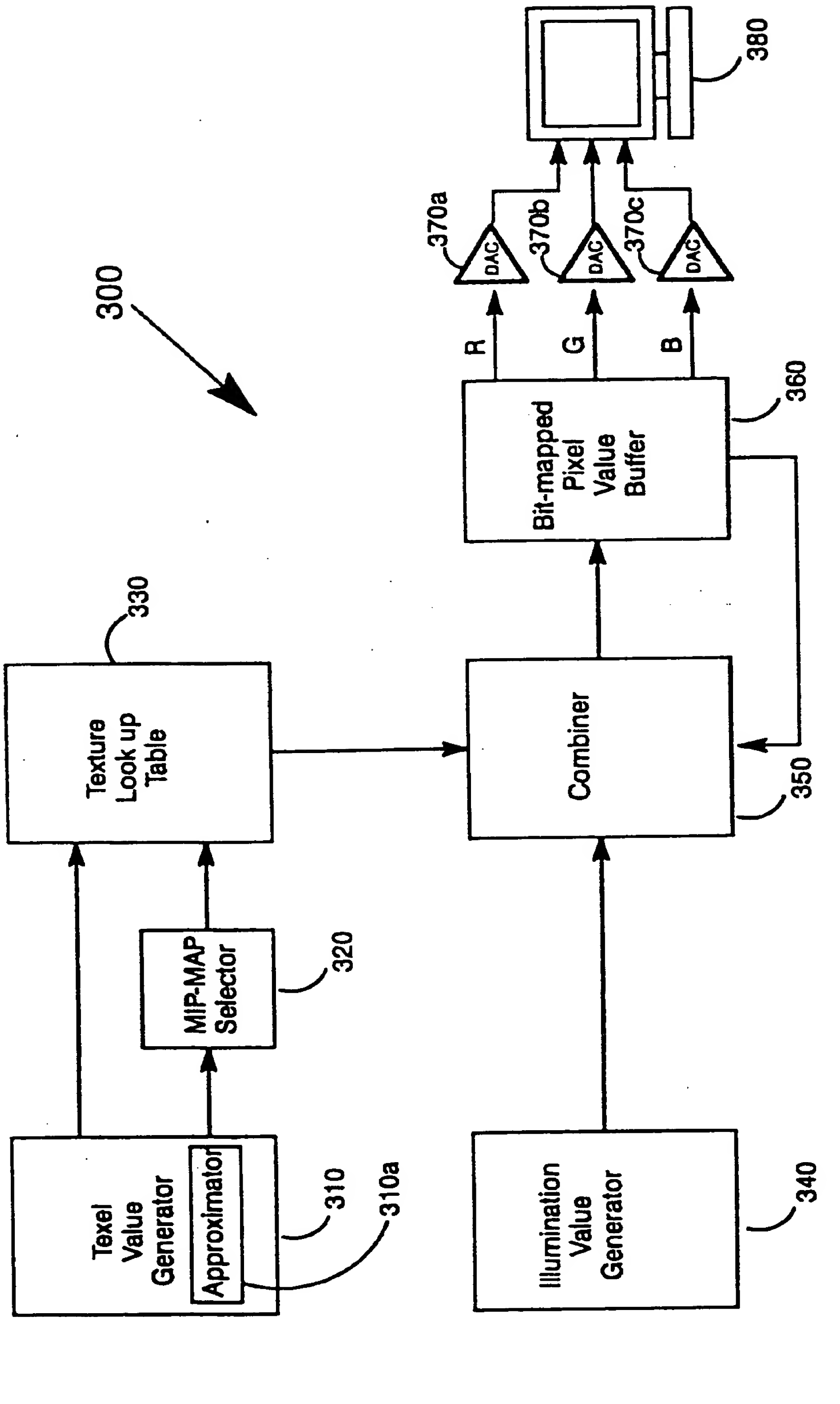

300
Texel Value Generator
Approximator
310
310a
MIP-MAP Selector
320
Texture Look up Table
330
Illumination Value Generator
340
Combiner
350
Bit-mapped Pixel Value Buffer
360
R
G
B
DAC
370a
DAC
370b
DAC
370c
380

FIG. 3

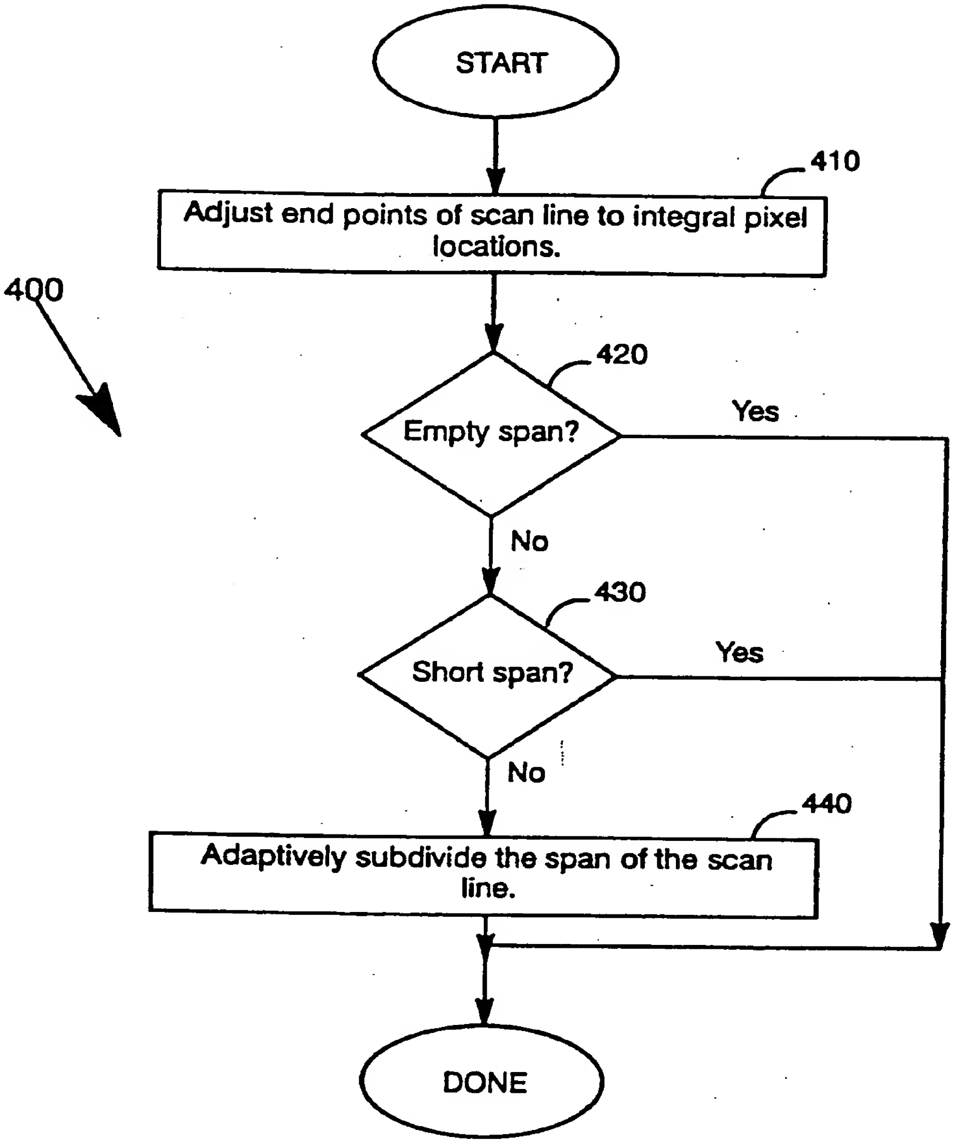

START
410
Adjust end points of scan line to integral pixel locations.
400
420
Empty span?
Yes
No
430
Short span?
Yes
No
440
Adaptively subdivide the span of the scan line.
DONE

FIG. 4A

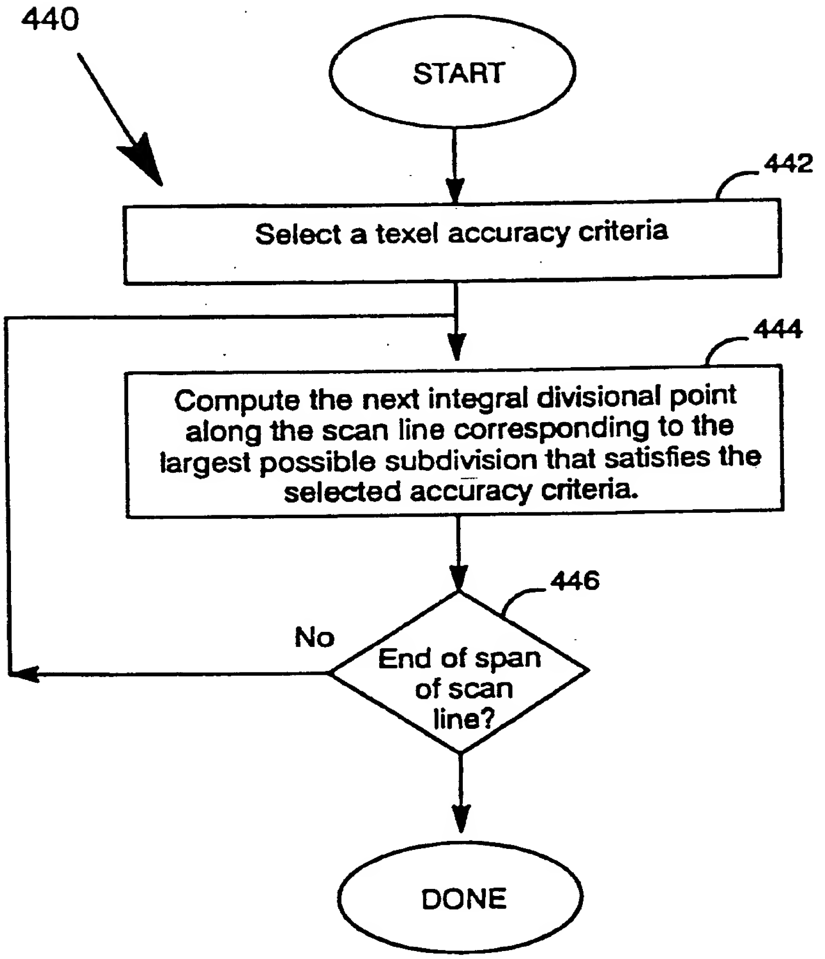

440
START
442
Select a texel accuracy criteria
444
Compute the next integral divisional point along the scan line corresponding to the largest possible subdivision that satisfies the selected accuracy criteria.
446
No
End of span of scan line?
DONE

FIG. 4B

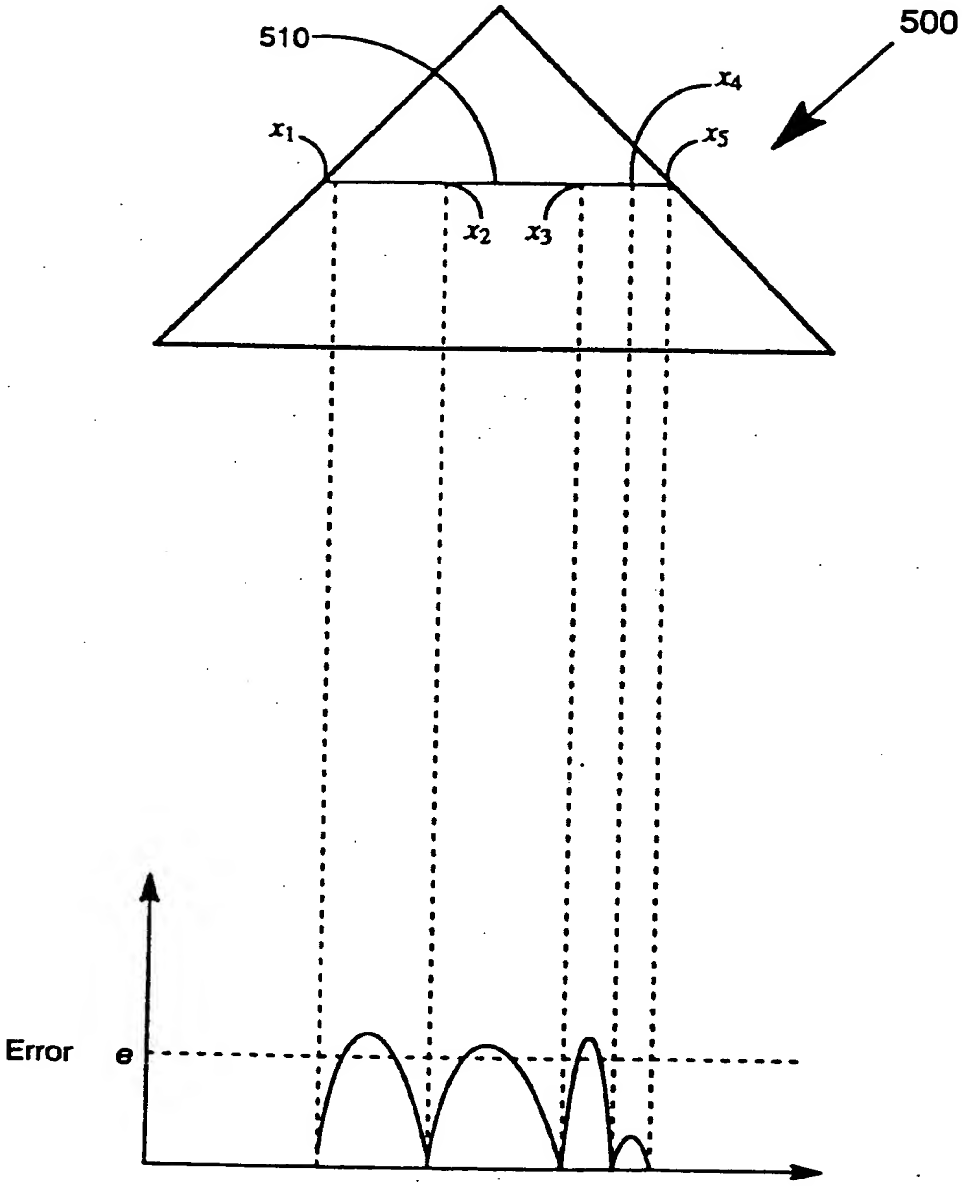


FIG. 5

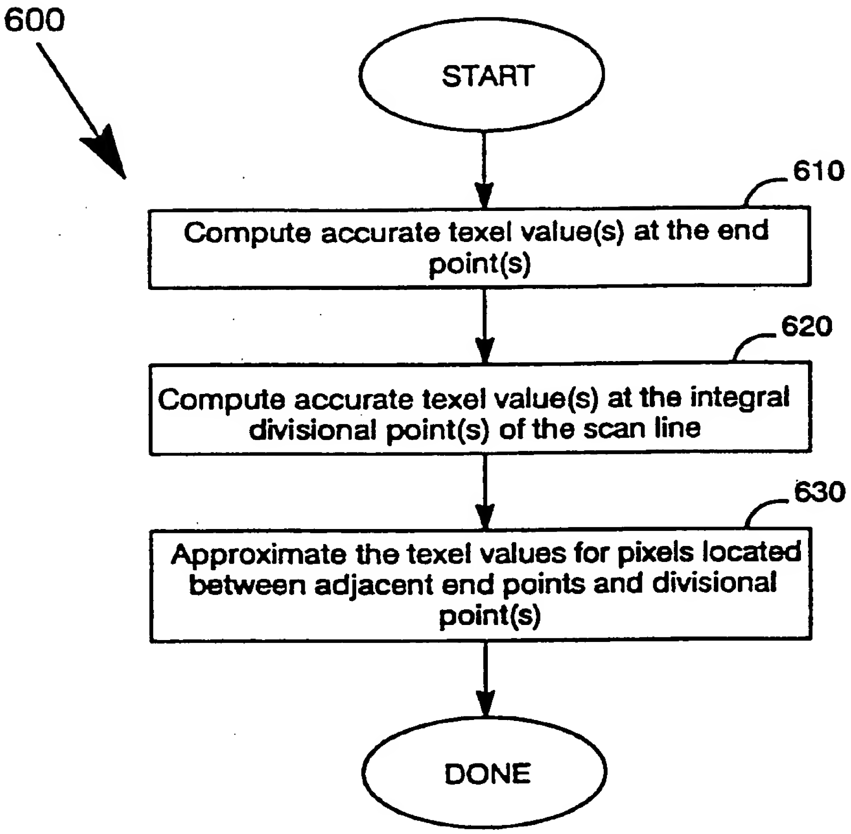

600
START
610
Compute accurate texel value(s) at the end point(s)
620
Compute accurate texel value(s) at the integral divisional point(s) of the scan line
630
Approximate the texel values for pixels located between adjacent end points and divisional point(s)
DONE

FIG. 6

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26.06.1996 Bulletin 1996/26

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(54) Perspective correction of texture in graphics by adaptive approximation

(57) The present invention provides a computer graphics system with a texel value generator capably of generating texel values using a minimal amount of computationally intensive divisions while maintaining a selectable texel accuracy criteria along a scan line. This is accomplished by adaptively selecting divisional points which delineate the scan line segments along each scan line such that the divisional points are as widely spaced as possible without exceeding the selected texel accuracy criteria. Having selected the texel accuracy criteria, such as a texel error bound optimally spaced, divisional points along the scan lines are selected as a function of the selected accuracy criteria. In general, since texture gradients are not evenly distributed over the surface of a given object and texture variations are present between different objects of the image, it is advantageous to adaptively select division points one at a time, skipping as many pixels in between divisional points as the local texture gradient will allow. Accurate texel values are computed at these divisional points and also at the end points of the scan line. Approximate texel values are then computed for the pixels located between adjacent pair of divisional points along the scan line using a suitable scheme such as linear interpolation.

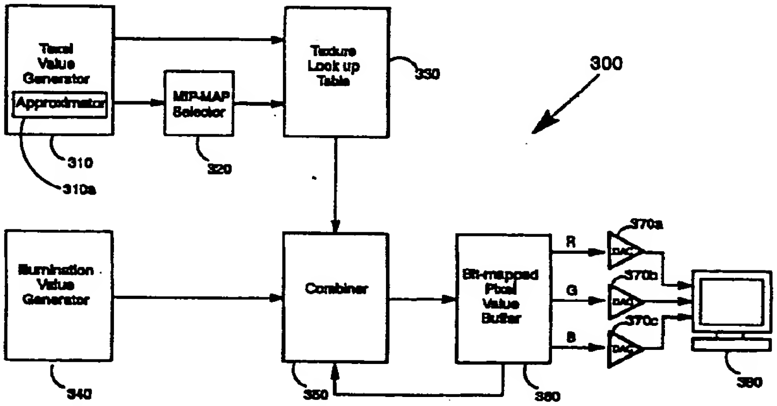

FIG. 3

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EUROPEAN SEARCH REPORT

Application Number
EP 95 30 9215

DOCUMENTS CONSIDERED TO BE RELEVANT

Category	Citation of document with indication, where appropriate, of relevant passages	Relevant to claim	CLASSIFICATION OF THE APPLICATION (Int.Cl.6)
A	US-A-5 307 450 (SILICON GRAPHICS) ---		G06T15/00 G06T15/10
A	EUROGRAPHICS, 4 - 8 September 1989, AMSTERDAM, pages 257-268, XP000132231 -----		
			TECHNICAL FIELDS SEARCHED (Int.Cl.6)
			G06T

The present search report has been drawn up for all claims

Place of search	Date of completion of the search	Examiner
THE HAGUE	12 June 1996	Burgaud, C

CATEGORY OF CITED DOCUMENTS

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